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CENTER FOR SYSTEMS RESEARCH

Multi-Input, Multi-Output Regulator Design for Constant Disturbances and Non-Zero Set Points with Application to Automatic Landing in a Crosswind

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August 1973

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ABSTRACT

Undesirable steady offsets result when a stationary, linear regulator using state feedback is subjected to constant disturbances and/or non-zero setpoints. To eliminate these offsets, the disturbances and non-zero setpoints can be fed forward to the control. Only when the number of outputs is less than or equal to the number of control inputs can the outputs be maintained at arbitrary non-zero setpoints.

The constant disturbances may be estimated from the state using an observer. An alternative is to feed back integrals of the deviation in the outputs; this amounts to a special form of disturbance observer and has the advantage that the steady performance is insensitive to small deviations in the system model parameters.

The state and the disturbance may be estimated using a constant gain Kalman filter. In this case we suggest that the constant disturbances be modeled as exponentially correlated processes with long correlation times. As an alternative, a Kalman filter (neglecting disturbances) can be used to estimate the state and the estimated state and integrals of the measured output deviation are then fed back to the control.

These results are applied to the problem of automatic landing of an aircraft in the presence of a steady crosswind. The goal of the control system is to maintain the lateral position and yaw attitude of the aircraft in alignment with the runway centerline.

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NOMENCLATURE

For the context of general results:

A	Symmetric, positive semi-definite weighting matrix on the states for quadratic synthesis
B	Symmetric, positive definite weighting matrix on the controls
C	Matrix of control gains. Subscripts refer to associated vector, e.g. C_x, C_y, etc.
F	State dynamics matrix
G	Control distribution matrix
H	Measurement distribution matrix
J	Performance index for an optimization problem
K	Matrix of filter or observer gains
L	A special matrix useful in integral control synthesis
M	Matrix for feed-forward control
P	Steady estimation error covariance matrix
Q	Power spectral density of process noise
R	Power spectral density of measurement noise
S	Solution matrix of the Ricatti equation for quadratic synthesis
$\mathscr{S}$	Diagonal matrix of eigenvalues of Euler-Lagrange system in quadratic synthesis. Subscript refers to sign of the real part.
t	Independent variable usually time or distance
T	Output distribution matrix
u	Control vector
v	Vector of integral control variables
w	Constant disturbance vector
x	State vector
X	Partition of the Euler-Lagrange eigenvector matrix corresponding to the state
y	Output vector
z	Measurement vector
Γ	Disturbance distribution matrix
η	Impulse or white noise process disturbance vector
λ, μ	Lagrange multiplier vectors

Λ Partition of the Euler-Lagrange eigenvector matrix corresponding to the adjoint variables

ν Measurement noise vector

χ Covariance matrix. Subscripts refer to associated vectors, e.g. $\chi_{xu} = E(xu^T)$

$(\dot{\ })$ Differentiation with respect to the independent variable

E( ) Expectation operator (mean value)

$(\hat{\ })$ Estimate of a vector

$(\tilde{\ })$ Error associated with the estimate of a vector, e.g. $\tilde{x}=\hat{x}-x$

$\triangleq$ Defined equal to

$(\ )^{-1}$ Inverse of a matrix

$(\ )^T$ Transpose of a matrix (or column vector)

$(\ )_d$ Desired value of a vector

$(\ )_s$ Steady value of a vector

For the context of specific results for the lateral motions of a DC-8 aircraft in landing approach:

V Reference airspeed of the aircraft (hft/sec , 1 hft = 100 ft)

w Steady crosswind (ft/sec)

x Independent variable = horizontal range (hft)

y Lateral position (ft)

δa Aileron (spoiler) deflection (deg)

δr Rudder deflection (deg)

ϵ Horizontal path azimuth angle (deg)

φ Roll angle (deg)

φ' Roll angle derivative (deg/hft)

ψ Yaw angle (deg)

ψ' Yaw angle derivative (deg/hft)

ω Turn rate (deg/sec)

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Chapter 1

Introduction

The purpose of a regulator is to hold certain outputs of a system near desired set point values in the presence of disturbances. Two types of disturbance commonly encountered are (1) impulse (initial condition) disturbances and (2) constant (step) disturbances. To counteract impulse disturbances, feedback regulators are used; they are designed by time domain, frequency response, and state variable quadratic synthesis techniques (B-2).

Constant disturbances arise from external influences on the system or from non-zero set points for the outputs. The effect of constant disturbances on systems with regulators designed for impulse disturbances is to drive them through a transient to some steady offset. Compensation for this offset is often possible by feeding the set points and external constant disturbances forward to the control. Di Caprio and Wang, 1969 (D-3), Bélanger, 1970 (B-1), Kwakernaak and Sivan, 1972 (K-2), and Power, 1973 (P-5) suggest breaking the non-zero set-point problem into two parts: (1) the determination of desired steady values of state and control which depend explicitly on the set points, and (2) the design of a regulator to control the deviations from the steady values. The similar problem of constant process disturbances is discussed in the scalar case by Athans, 1971 (A-2), and in the vector case by Johnson, 1971 (J-2). However, both of these treatments attempt to bring all of the state components to zero in the steady-state, which is usually not possible or desirable. In addition, the steady-state solution for zero output offset, when it exists, is often not unique. A static optimization procedure presented herein removes this ambiguity.

If the constant disturbances are not measured directly, they may still be estimated (or otherwise accounted for) using only the available measurements.

For single-input, single-output systems, a well-known compensation technique for constant disturbances is the addition of integral control

(C-1 and D-1). This consists of feeding back the first integral of the deviation in the desired output. Through the application of frequency or time domain analysis, the integral control gain is adjusted for a suitable transient behavior. For a stable system, such integral feedback causes the output being integrated to be driven to a zero steady offset.

In the case of multi-input, multi-output systems, an observer (L-2 and B-3) can often be designed to estimate the disturbance. The estimate is then fed to the control input. Johnson, 1971 (J-2) suggests this approach, although his formulation is somewhat different from the approach presented herein.

The concept of integral control and its zero-offset properties can be extended to multi-variable systems. Johnson, 1968 (J-1), Latour, 1971 (L-1), Athans, 1971 (A-2), and Anderson and Moore, 1971 (A-1) suggest modifying the performance index of the linear regulator, quadratic synthesis technique to include weighting on the derivative of the control input. After algebraic manipulation, the resulting control law is found to include integration of linear combinations of the state variables. One problem with this approach is that these combinations of state variables do not generally correspond to the desired outputs. Hence, the outputs will be driven to some offset unless the control can directly cancel the effect of the constant disturbance. This latter condition is overly restrictive. Porter, 1971 (P-1), Bryson, 1972 (B-5), and Kwakernaak and Sivan, 1972 (K-2) suggest the more fruitful approach of adding to the performance index weights on the additional states formed by integration of the outputs. Meyer, Martin, and Power, 1973 (M-1) present an interesting alternative formulation by weighting the output deviation and state derivative in the performance index. Some caution must be taken, however, since adding additional states may make a previously controllable system uncontrollable. Power and Porter, 1970 (P-3) give the requisite controllability conditions for the case of additional integrated states. These conditions are found to be related to the ability to find some control that will yield zero offset. A

technique is presented herein which allows the use of the set-point gains for zero offset to augment a stable state feedback regulator with integrals of the output. The closed-loop poles of the overall system are those of the original regulator plus additional poles, corresponding to the integrations, with an arbitrary degree of stability. This technique requires very little additional computation and facilitates on-line switching from proportional to proportional-integral control modes. While preparing this report, reference was found to a similar pole assignment technique (P-4). The relation between the integral control and observer formulations is also explored. Davison and Smith, 1971 (D-2) show that feedback of the state plus integrals of the output constitutes a minimal order realization of a system, operating on the state, for which the overall closed-loop poles can be assigned arbitarily and the steady output is zero. It is further shown here that integral control amounts to a special form of observer.

In many cases, the full state is not known exactly. Instead, only certain noisy measurements are available. When an optimal filter (K-1, B-2) is used to estimate the state and constant disturbances, it is found that the asymptotic estimates of the constant disturbances are exact. Thus, the steady filter will have zero gains and cannot be used. To circumvent this problem, an observer design technique can be used. Kwatny, 1972 (K-3) found that by modeling the disturbance as a random walk process (integral of a white-noise process), such observers can effectively be designed. Alternatively, a filter which estimates the state alone (ignoring the constant disturbance) can be used in conjunction with an integral control law, providing the desired outputs are directly measured. This latter technique has the advantage that the stationary output covariance does not depend upon the nature of the constant disturbance.

Each of the above topics is discussed in detail in the remaining sections, and the results are applied to the problem of designing a lateral autopilot for automatic aircraft landing in the presence of a steady crosswind.

Chapter 2

Review of Regulator Design for Impulse Disturbances with Exact Knowledge of the State

2.1 - System Model

The systems considered here can be represented by a set of constant coefficient linear differential equations

$$\dot{x} = Fx + Gu + \Gamma\eta \tag{2.1.1}$$

where
- x = state vector
- u = control input vector
- η = disturbance vector
- F = state dynamics matrix
- G = control distribution matrix
- Γ = disturbance distribution matrix
- $(\dot{\ })$ denotes derivative with respect to the independent variable (usually time or distance)

The objective of regulator design is to keep the state, x, near zero in the presence of disturbances, η. Here we assume that (a) the disturbances are non-zero over periods of time short compared to response times of the system (impulse type disturbances), (b) the time average of the disturbances is zero (positive impulses are balanced by negative impulses), and (c) otherwise, the disturbances are unpredictable. Since the disturbances are unpredictable except for the zero average, the best thing to do is to assume $\eta=0$ in the future and design the regulator to bring the state to zero from arbitrary initial conditions in an acceptably short period of time. Two techniques for designing such a control law are presented in the following sections.

2.2 - Pole Assignment (B-3)

For the linear system given by equation (2.1.1), let the control law be defined

$$u \stackrel{\Delta}{=} -Cx \tag{2.2.1}$$

Thus, the closed-loop system will be given by

$$\dot{x} = (F-GC)x \ , \quad x(t_o) = x_o \tag{2.2.2}$$

The transient response will be determined by the eigenvalues (closed-loop poles) of the (F-GC) matrix. Stability is assured if all the eigenvalues have negative real parts. The dominant transient characteristics will be determined by the poles with the smallest magnitudes. If the system $[F:G]$ is controllable, the eigenvalues can be arbitrarily assigned (complex values occurring in conjugate pairs) through the choice of C. To see this, consider the characteristic equation given by

$$|sI - F + GC| = 0 \tag{2.2.3}$$

This determinant is a polynomial in the variable s and hence will have coefficients which are functions of the various elements of F, G, and C. Selection of the poles (roots of 2.2.3) prescribes these coefficients. Equating the coefficients yields algebraic (generally non-linear) equations to be satisfied by the components of C. In the case of a single input system there are exactly enough components of C so the solution will be unique. However, for multi-input systems the solution for C is generally non-unique, leading to one of the difficulties with the method.

Care must also be taken when assigning the magnitude of the eigenvalues. Generally, if the magnitude of the closed-loop eigenvalues greatly exceeds the magnitude of the open-loop eigenvalues, large components of the control gain matrix (C) will be required. Large gains may lead to control saturation and noise amplification problems.

2.3 - Quadratic Synthesis (B-2)

Consider the following optimal regulator problem. We want to minimize with respect to control (u) the performance index

$$J = 1/2 \int_{t_o}^{\infty} (x^T A x + u^T B u) dt \tag{2.3.1}$$

subject to the differential equation constraint

$$\dot{x} = Fx + Gu \ .$$

The matrices A and B are chosen subjectively to weigh the output deviation against control magnitude. The solution is the linear feedback law

$$u = -Cx \tag{2.3.2}$$

where $$C = B^{-1}G^{T}S \quad , \quad SF + F^{T}S + A - SGB^{-1}G^{T}S = 0 \ . \tag{2.3.3}$$

The solution of this non-linear Ricatti equation for the matrix (S) can be obtained efficiently by eigenvalue decomposition of the Euler-Language system (B-6).

$$\begin{bmatrix} F \ , & -GB^{-1}G^{T} \\ -A \ , & -F^{T} \end{bmatrix} \begin{bmatrix} X_{+}X_{-} \\ \Lambda_{+}\Lambda_{-} \end{bmatrix} = \begin{bmatrix} X_{+}X_{-} \\ \Lambda_{+}\Lambda_{-} \end{bmatrix} \begin{bmatrix} \mathcal{S}_{+} & 0 \\ 0 & -\mathcal{S}_{+} \end{bmatrix} \tag{2.3.4}$$

The matrix (S) is given in terms of the partitioned eigenvectors as

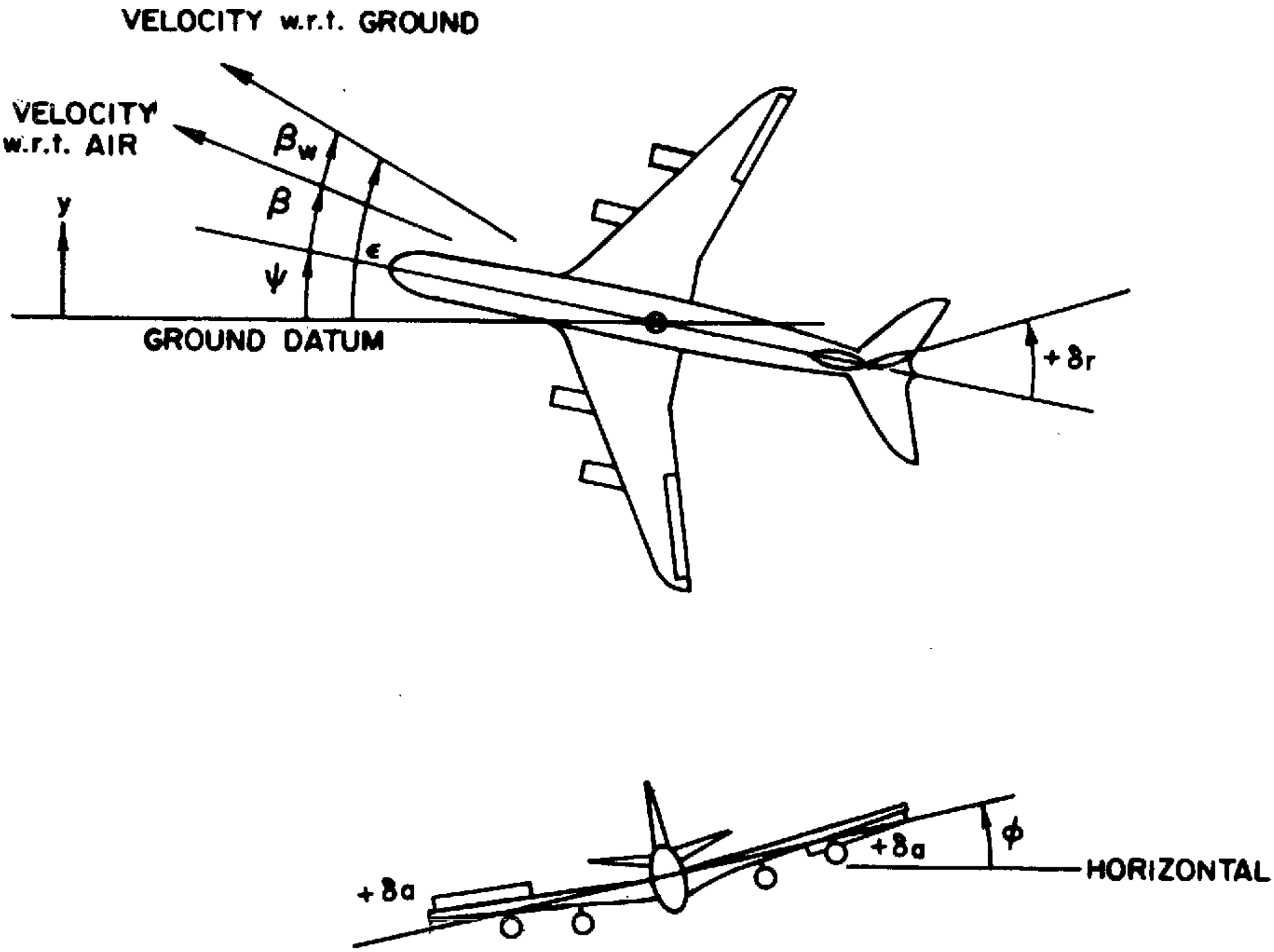
$$S = \Lambda_{-}X_{-}^{-1} \tag{2.3.5}$$

2.4 - Example: A Lateral Aircraft Autopilot

Consider the design of an autopilot to align an aircraft with the runway in the final phase of the approach and landing. The lateral motions of an aircraft are well-approximated by a sixth order linear model (Appendices A&B). The model states are roll attitude (φ) and derivative (φ') yaw attitude (ψ) and derivative (ψ'), horizontal path angle (ϵ), and lateral position (y) (See Fig. 1). The controls are the aileron (δa) and rudder (δr) deflections. The equations are written in terms of the horizontal range (x) as the independent variable, and the F,G matrices for a DC-8 aircraft in landing approach are given in Appendix B.

Applying the quadratic synthesis technique described in the previous section, a performance index is chosen which weighs deviations in yaw attitude and lateral position against the control deflections.

Definition of Lateral Aircraft States



β = sideslip w.r.t. the air (gives the aerodynamic forces)

β_w = non-dimensional side wind (w/V)

φ = roll attitude

ψ = yaw attitude

ϵ = flight path azimuth angle ($\dot{y}/V$)

y = lateral position

x = range

δa = aileron-spoiler deflection

δr = rudder deflection

FIG. 1

$$J = 1/2 \int_0^\infty \left[(y/a_1)^2 + (\psi/a_2)^2 + (\delta a/b_1)^2 + (\delta r/b_2)^2 \right] dx \tag{2.4.1}$$

Choosing the weighting factors:

$a_1 = 15$ ft, $a_2 = 2$ deg, $b_1 = 10$ deg, and $b_2 = 10$ deg

yields the feedback control law (see Appendix B):

$$\left.\begin{aligned} \delta a &= -3.571\varphi' - 2.471\varphi - 4.630\psi' - 1.602\psi - 8.546\epsilon - 0.6592y \\ \delta r &= -0.2328\varphi' - 0.4959\varphi - 9.187\psi' - 3.627\psi - 2.917\epsilon - 0.0996y \end{aligned}\right\} \tag{2.4.2}$$

where: δa, δr, φ, ψ, ϵ are taken in units of deg.

y is taken in units of ft.

φ', ψ' are taken in units of deg/hft*.

Using this control law, the closed-loop poles are shown in Fig. 2. The lateral position and yaw attitude are taken as zero when the aircraft is aligned with the extended runway centerline. The transient response to initial conditions of φ=5 deg, ψ=5 deg, and y=15 ft is shown in Fig. 3.

*1hft = 100 ft

Open and Closed Loop Poles for

State Feedback Lateral Control System

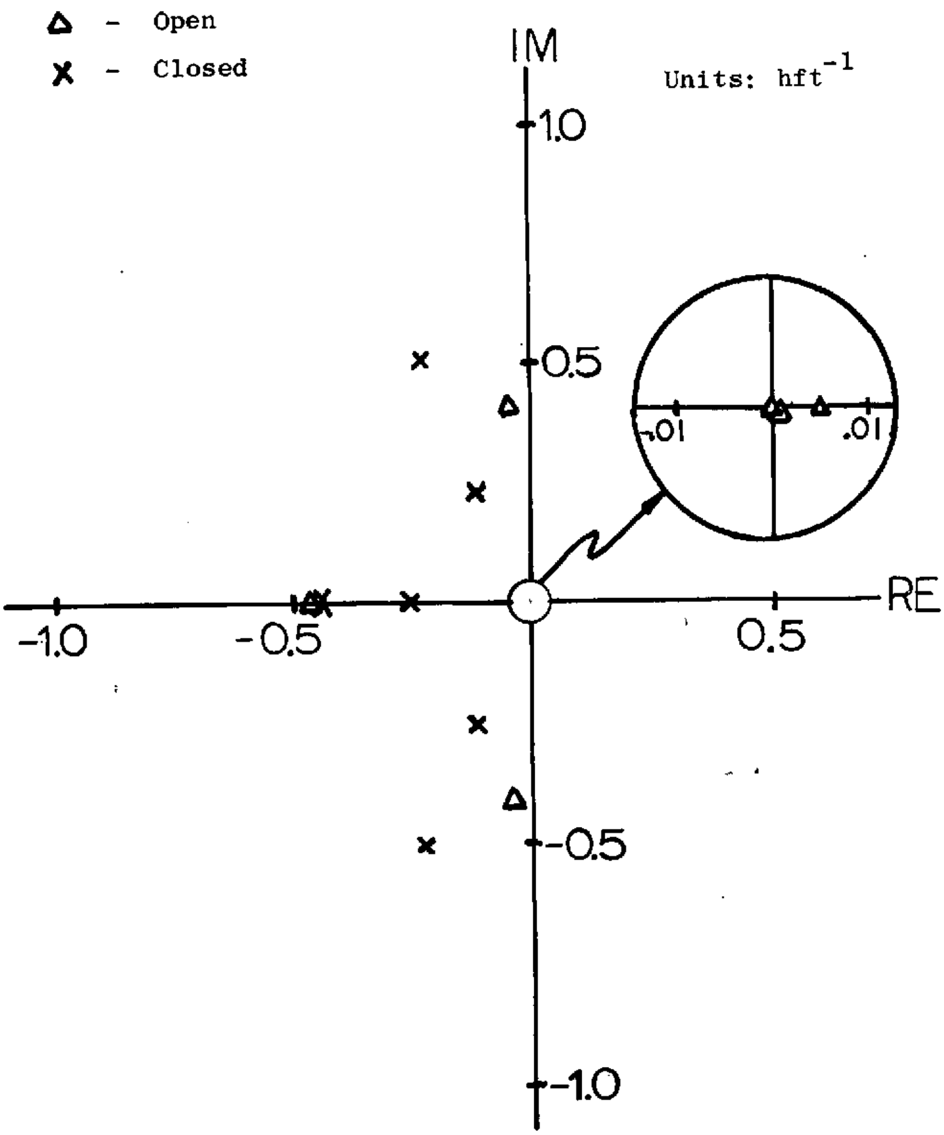


FIG. 2

Response to Initial Condition Disturbance

Using State Feedback Control

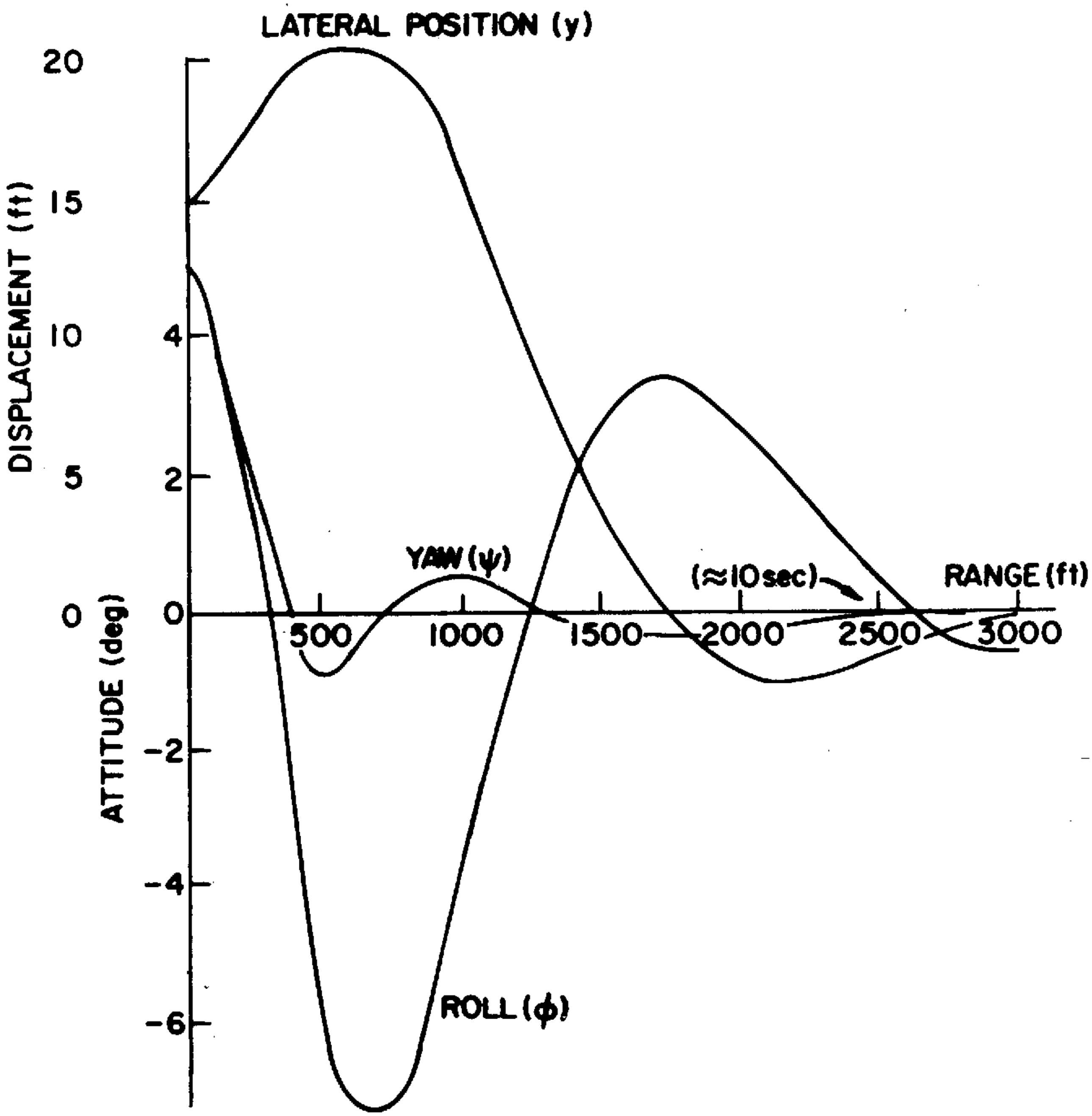


FIG. 3

Chapter 3

Regulator Design for Constant Disturbances and Non-Zero Set Points with Exact Knowledge of the State

3.1 - The Effect of Uncompensated Constant Disturbances

Consider the linear system model of Chapter 2 with the addition of a constant disturbance vector and a specified output vector:

$$\dot{x} = Fx + Gu + \Gamma w \quad , \quad y = Tx, \tag{3.1.1}$$

where w = constant disturbance vector,

Γ = constant disturbance distribution matrix,

y = output vector,

T = output distribution matrix.

The control objective is to bring and hold the outputs (y) to the set points (y_d).

To illustrate the effect of a constant disturbance which is not compensated for, again consider the lateral aircraft control example. Using the feedback control law given before, the effect of a steady crosswind is shown in Fig. 4. The initial conditions are taken as zero, which means that the aircraft, initially in trim, suddenly encounters a crosswind which is then constant. The crosswind speed is taken as 25 ft/sec (≈ 15 kts). Note that the lateral position (y), the yaw attitude (ψ), and roll attitude (φ) all go to appreciable steady offsets.

3.2 - Design with Exact Knowledge of the Disturbances

As a first step in the development, suppose that the disturbances (w) are known exactly. A reasonable control law would then be to feedback the deviations in the state from a desired steady value (D-3, K-2, P-5). Thus,

$$u = u_s - C_x \, (x-x_s) \tag{3.2.1}$$

The desired steady values u_s and x_s are chosen to satisfy the steady-state and zero-offset constraints and thus depend upon the set point and disturbances.

Response to Step Crosswind Using State Feedback Only

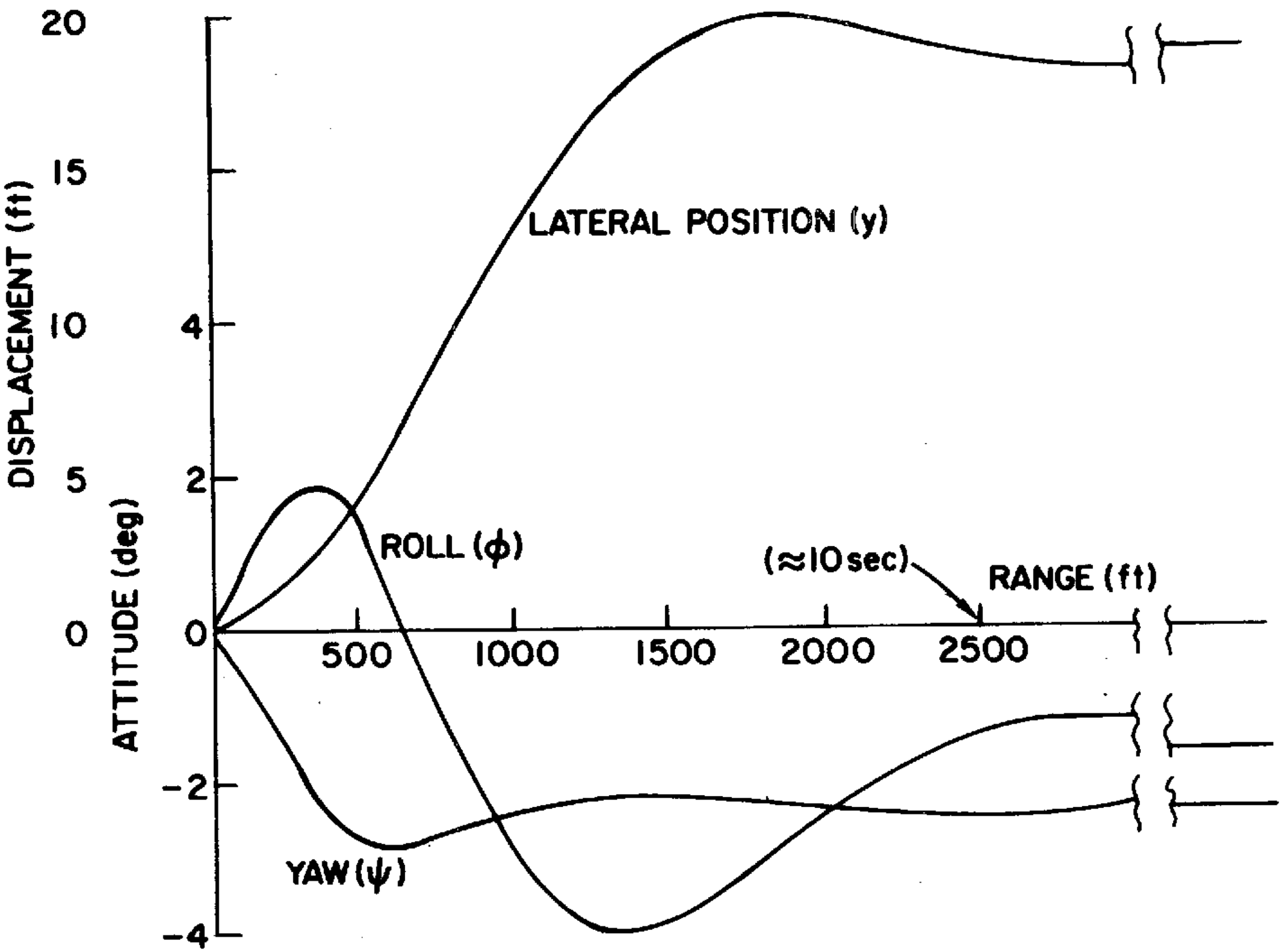


FIG. 4

$$Fx_s + Gu_s + \Gamma w = 0 \tag{3.2.2}$$

$$Tx_s - y_d = 0 \tag{3.2.3}$$

When the control law (3.2.1) is used in the system equations (3.1.1), the result is

$$\dot{x} = Fx + G[u_s - C_x(x-x_s)] + \Gamma w \tag{3.2.4}$$

or

$$\frac{d}{dt}(x-x_s) = F(x-x_s) + G(u-u_s) + Fx_s + Gu_s + \Gamma w$$

$$= F(x-x_s) + G(u-u_s) \tag{3.2.5}$$

Thus, the problem of choosing the gain C_x is exactly that of the regulator problem for impulse disturbances of Chapter 2, where the deviations from steady values are used as state and control. The remaining problem is then the choice of appropriate steady values x_s and u_s.

The linear equations (3.2.2) and (3.2.3) have a solution if the rank of the matrix $\begin{bmatrix} F,G \\ T,0 \end{bmatrix}$ is equal to the rank of the column-augmented matrix

$$\begin{bmatrix} F & , & G & , & \Gamma w \\ T & , & 0 & , & y_d \end{bmatrix}$$

For arbitrary non-zero w and y_d this requires that the number of outputs be less than or equal to the number of controls, and that the rank of $\begin{bmatrix} F,G \\ T,0 \end{bmatrix}$ be equal to the number of rows.

When the number of outputs equals the number of controls the solution is unique.

$$\begin{bmatrix} x_s \\ u_s \end{bmatrix} = \begin{bmatrix} M_{xw} & , & M_{xy} \\ M_{uw} & , & M_{uy} \end{bmatrix} \begin{bmatrix} -\Gamma w \\ y_d \end{bmatrix} \tag{3.2.6}$$

where

$$\begin{bmatrix} M_{xw} , M_{xy} \\ M_{uw} , M_{uy} \end{bmatrix} = \begin{bmatrix} F , G \\ T , 0 \end{bmatrix}^{-1} \tag{3.2.7}$$

When the number of outputs is less than the number of controls, and the matrix $\begin{bmatrix} F,G \\ T,0 \end{bmatrix}$ has full rank, an infinite number of solutions exist. A useful technique for specifying a solution is to minimize the performance index

$$J = 1/2(x_s^T A x_s + u_s^T B u_s) \tag{3.2.8}$$

subject to the steady-state and zero-offset constraints (3.2.2) and (3.2.3). This yields unique solutions of the same form as (3.2.6). Determination of the gain matrices in (3.2.6) for this case is discussed in Appendix C.

Using (3.2.6) we may rewrite (3.2.1) in the form

$$u = -C_x x + C_y y_d - C_w w \ , \tag{3.2.9}$$

where

$$C_y = M_{uy} + C_x M_{xy} \quad , \quad C_w = (M_{uw} + C_x M_{xw})\Gamma \ .$$

For the lateral aircraft control problem, there are two outputs, yaw attitude and lateral position, and two controls, rudder and aileron deflection. The choice of outputs was made among the three possibilities of lateral position, yaw attitude, and roll attitude. Obviously, the lateral position is of primary importance for an automatic landing. Deviations in roll atttitude will cause touchdown on one landing gear, while deviations in yaw attitude result in side skidding. The side skidding is deemed the more dangerous situation.

Applying the previous results then yields the additional gains for non-zero set points and the steady crosswind. The control law becomes

$$\begin{aligned}\delta a &= -3.571\varphi' - 2.471\varphi - 4.630\psi' - 1.602\psi - 0.843\psi d - 8.546\epsilon \\ &\quad - 0.6592(y-y_d) - 0.5732w \\ \delta r &= -0.2328\varphi' - 0.4959\varphi - 9.187\psi' - 3.622\psi + 5.52\psi d - 2.917\epsilon \\ &\quad - 0.0996(y-y_d) + 0.4461w\end{aligned} \tag{3.2.10}$$

where all quantities are as before except w which is the steady crosswind in units of ft/sec.

The transient response to zero initial conditions is shown in Fig. 5 for $y_d = \psi_d = 0$. Note that both the lateral position and yaw attitude deviations go to zero in the steady state, while roll attitude maintains some offset. This offset is given as

$$\varphi_s \approx (-0.108 \text{ deg-sec/ft})w \tag{3.2.11}$$

3.3 - Design Using Estimates of the Disturbances

The compensation method just described assumes perfect knowledge of the disturbances. If the disturbances cannot be measured directly it may still be possible to estimate them. One method of doing this is to synthesize a reduced order observer to estimate the disturbances (L-2, J-2).

Since the state is assumed to be known exactly, its derivative, in principle, is also known exactly. Thus, let an estimate of w be defined as:

$$\dot{\hat{w}} = K[(Fx+Gu+\Gamma\hat{w}) - \dot{x}] \quad , \quad \hat{w}(t_o) = 0 \tag{3.3.1}$$

where K is a constant matrix to be chosen. The estimate error, $\tilde{w} \triangleq \hat{w} - w$, is then given by

$$\dot{\tilde{w}} = K\Gamma\tilde{w} \tag{3.3.2}$$

If the matrix K is chosen so that $K\Gamma$ is negative definite, the estimate error ($\tilde{w}$) will go to zero asymptotically, i.e. $\hat{w} \to w$.

The control used is the obvious modification of (3.2.9):

$$u = -C_x x + C_y y_d - C_w \hat{w} \tag{3.3.3}$$

Controlled Response to Step Crosswind

Using Exact Knowledge of State and Wind

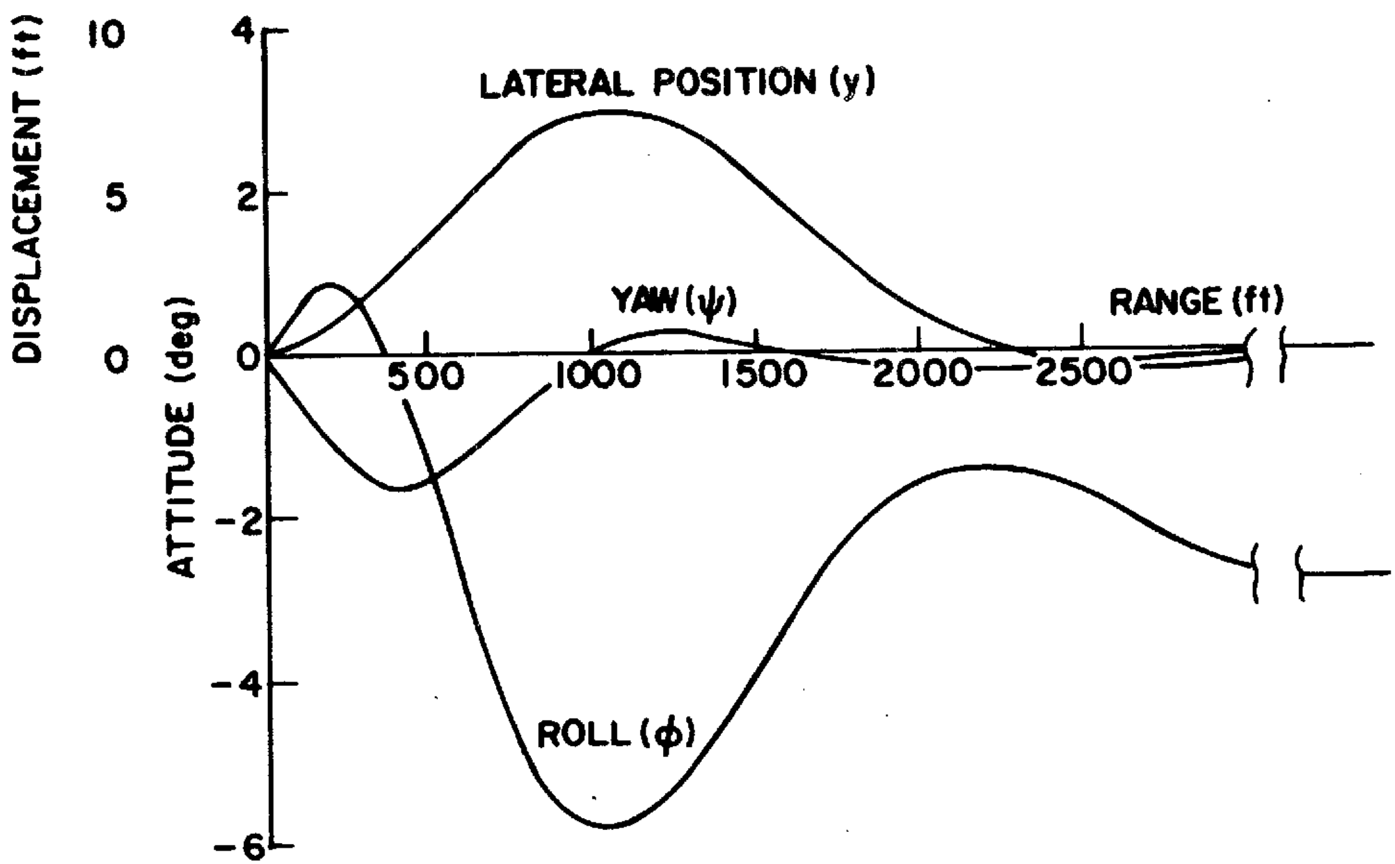


FIG. 5

Since $\hat{w} = w+\tilde{w}$, and $\tilde{w}$ does <u>not</u> depend on x (Eq. 3.3.2) the eigenvalues of the controlled system are the eigenvalues of $F-GC_x$ and of $K\Gamma$.

To avoid differentiating the state in (3.3.1), define $w_* \overset{\Delta}{=} \hat{w} + Kx$; then

$$\left.\begin{aligned} \dot{w}_* &= K(Fx+Gu+\Gamma\hat{w}), \\ \hat{w} &= w_* - Kx . \end{aligned}\right\} \tag{3.3.4}$$

The deficiency of this method of compensation is that it depends upon precise knowledge of the system model. If the actual F, G, and Γ matrices differ from the estimated ones, a non-zero steady offset results.

As an example, consider again the lateral aircraft control problem where the disturbance is the steady crosswind. Let the observer gains (K) be chosen so that $K=-\sigma(\Gamma^T\Gamma)^{-1}\Gamma^T$. Thus, the observer characteristic matrix ($K\Gamma$) becomes the scalar

$$K\Gamma = -\sigma \tag{3.3.5}$$

and the observer pole is given by $-\sigma$. In this case, σ was chosen as 0.3 hft^{-1} , which lies near the middle of the closed-loop regulator poles (see Fig. 2). The response to a 25 ft/sec crosswind is shown in Fig. 6. The steady values of lateral position (y) and yaw attitude (ψ) are still zero, but the transient is somewhat more pronounced than in the case of exact disturbance feedback. This is due to the lag in estimating the steady crosswind.

3.4 - Design Using Estimates of the Output Deviations Produced by Uncompensated Constant Disturbances (Integral Control)

An alternative to estimating the disturbances is to estimate the steady output deviations that would be produced by constant disturbances if the system were not compensated. As we shall show, this leads directly to a generalization of classical integral control.

Controlled Response to Step Crosswind

Using Exact State and Estimated Wind

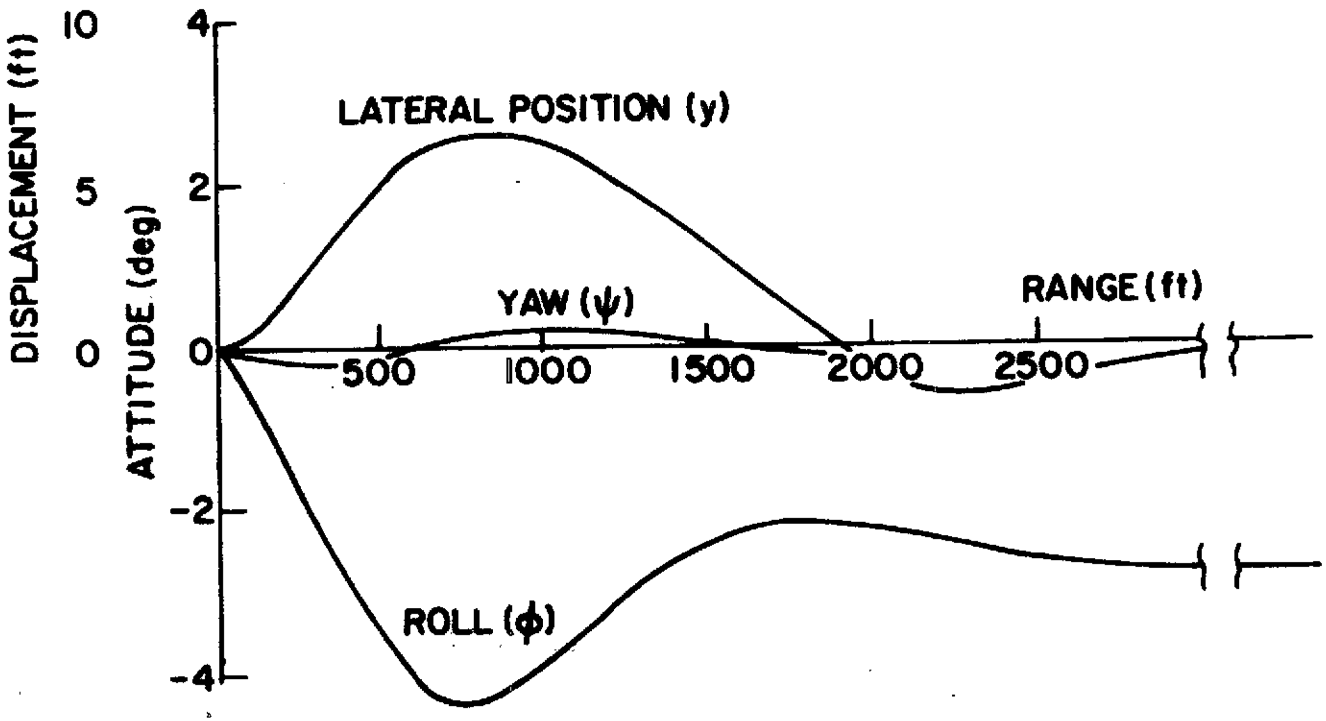


FIG. 6

Let

$$u = -C_x x + C_y y_o \tag{3.4.1}$$

Steady-state solutions, if any exist, must satisfy

$$\left.\begin{aligned} 0 &= (F-GC_x)x_s + GC_y y_o + \Gamma w , \\ y_d &= T x_s . \end{aligned}\right\} \tag{3.4.2}$$

If the system is controllable, $F-GC_x$ is negative-definite, so

$$x_s = -(F-GC_x)^{-1}(GC_y y_o + \Gamma w) \tag{3.4.3}$$

Substituting (3.4.3) into the second set of equations in (3.4.2) gives

$$y_d - L\Gamma w = LGC_y y_o \tag{3.4.4}$$

where $$L \triangleq -T(F-GC_x)^{-1} .$$

Equation (3.4.4) is satisfied for all y_d and w if

$$\begin{aligned} LGC_y &= I , \\ y_o &= y_d - L\Gamma w . \end{aligned} \tag{3.4.5}$$

Gains C_y can be found to satisfy (3.4.5) provided the number of outputs is less than or equal to the number of controls (and LG of full rank).

An estimate of $y_w \triangleq L\Gamma w$ can be obtained from

$$\dot{\hat{y}}_w = KL(\dot{x}-Fx-Gu-\Gamma\hat{w}) \tag{3.4.6}$$

$$= -K(\hat{y}_w - y_w) \tag{3.4.7}$$

Since $\dot{y}_w = 0$, it follows from (3.4.7) that

$$\dot{\tilde{y}}_w = -K\tilde{y}_w \quad \text{where} \quad \tilde{y}_w \triangleq \hat{y}_w - y_w . \tag{3.4.8}$$

Thus if K is chosen to be positive-definite $\tilde{y}_w \to 0$. Substituting (3.4.1) into (3.4.6) yields

$$\dot{\hat{y}}_w = KL\dot{x} - KL[(F-GC_x)x + GC_y y_o - GC_w\hat{w} + \Gamma\hat{w}] \tag{3.4.9}$$

Using (3.4.5) and $L(F-GC_x) = -T$, Eqn. (3.4.9) becomes

$$\dot{\hat{y}}_w = KL\dot{x} - K(-Tx+y_d) \tag{3.4.10}$$

To avoid differentiating x , define $y_* \triangleq \hat{y}_w - KLx$; then

$$\left.\begin{aligned} \dot{y}_* &= K(Tx-y_d) , \qquad y_*(t_o) = -KLx(t_o) , \\ \hat{y}_w &= y_* + KLx , \\ \text{where} \qquad u &= -C_x x + C_y(y_d-\hat{y}_w) \end{aligned}\right\} \tag{3.4.11}$$

Now (3.4.11) is obviously a generalization of "integral control" from the single-input, single-output case to the multi-input, multi-output case. In steady-state, $\dot{y}_*=0 \Rightarrow Tx \to y_d$, even if F, G, Γ were not estimated precisely.

Since (3.4.8) does not depend on x , it follows that the closed-loop eigenvalues of the system using (3.4.11) are the eigenvalues of $F-GC_x$ plus the eigenvalues of $-K$, which are determined separately.

A slightly more familiar form of integral control is obtained from (3.4.11) by introducing a vector v such that

$$y_* \triangleq Kv \tag{3.4.12}$$

Then (3.4.11) may be written as:

$$\begin{aligned} \dot{v} &= Tx - y_d , \qquad v(t_o) = -Lx(t_o) , \\ u &= -(C_x+C_yKL)x + C_y y_d - C_y Kv , \end{aligned} \tag{3.4.13}$$

where $LGC_y = I$.

The eigenvalues of the closed-loop system are still the eigenvalues of $F-GC_x$ and of $-K$. The estimated steady output deviations that would be produced by constant disturbances if the system were not compensated are

$$\hat{y}_w = K(v+Lx) . \tag{3.4.14}$$

If integral feedback is added to a system already designed with state feedback without modifying the state feedback gains (note C_x is modified to C_x+C_yKL in (3.4.13)), the transient behavior of the resulting closed-loop system is usually degraded (occasionally the system is even made unstable). Obviously, it is possible to design the state feedback gains simultaneously with the selection of the integral feedback gains so that the transient behavior is acceptable. This can be done, for example, by quadratic synthesis (see Appendix D).

A satisfactory state-plus-integral control system can always be found provided:

(a) The system (F,G) is controllable.

(b) The number of outputs is less than or equal to the number of control inputs.

(c) The matrix $\begin{bmatrix} F,G \\ T,0 \end{bmatrix}$ has full rank, or, equivalently, the matrix LG has rank equal to the number of outputs.

The major advantage of integral control is its relative insensitivity to errors in the system model. If the actual F, G, and Γ matrices differ from those used in the design, the transient behavior and the steady-state values of x and u are affected. However, unless the system is actually destabilized, the steady-output offset will still be zero.

Again, consider the lateral aircraft control example. Since it is desired to drive both the outputs, lateral position (y) and yaw attitude (ψ), to zero offset, the feed forward gains will be given by (3.4.5) as:

$$C_y = (LG)^{-1} = \begin{bmatrix} .659 , & -.843 \\ .100 , & 5.52 \end{bmatrix} \qquad (3.4.15)$$

Thus, for $K = \begin{bmatrix} \sigma,0 \\ 0,\sigma \end{bmatrix}$ and $\sigma = 0.3$ hft^{-1} the additional integral control poles will be located at the same place as the observer pole of Section 3.3. The resulting feedback control law is

$$\delta a = -5.796\varphi' - 4.567\varphi - 7.703\psi' - 2.768\psi - 22.21\epsilon - 2.547y - 0.843\psi_d$$
$$+ 0.659y_d - 0.1978 \int (y-y_d)dx - 0.2502 \int (\psi-\psi_d)dx$$
$$\delta r = 0.1203\varphi' - 0.6733\varphi - 14.01\psi' - 6.995\psi - 5.658\epsilon - 0.2750y$$
$$+ 5.52\psi_d + 0.100y_d - .0298 \int (y-y_d)dx - 1.657 \int (\psi-\psi_d)dx$$
(3.4.16)

The transient response to a 25 ft/sec steady crosswind from zero initial conditions with $\psi_d = y_d = 0$ is shown in Fig. 7. The steady offset for lateral position (y) and yaw attitude (ψ) is again zero, and the transient response is almost identical to that of the first order reduced observer of Section 3.3 where the disturbance itself was estimated.

Controlled Response to Step Crosswind

Using Exact State and Integral Feedback on y and ψ

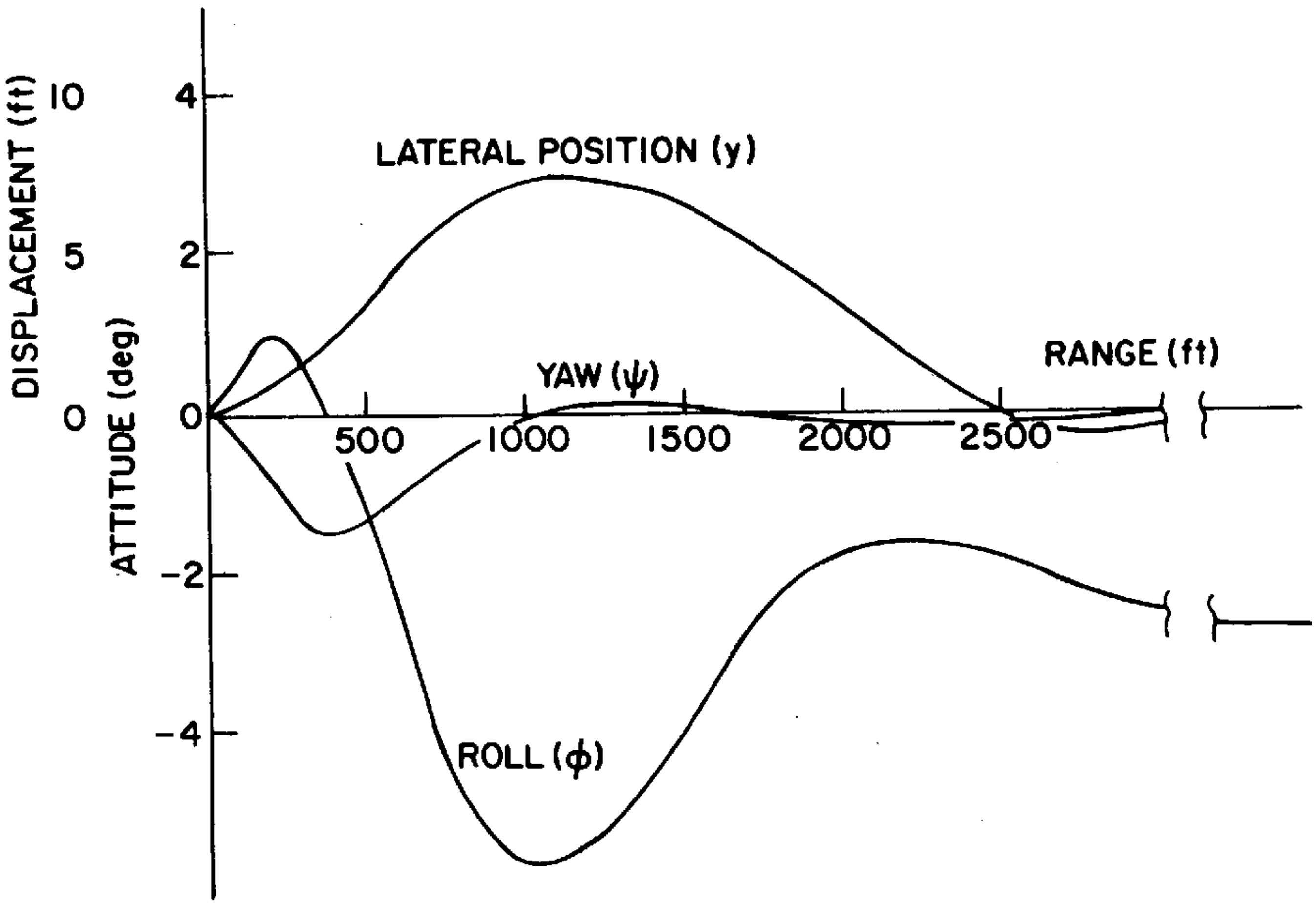


FIG. 7

Chapter 4

Review of Regulator Design for White Noise Disturbances Using Estimates of the State

4.1 - System Model

It is exceptional to be able to measure all of the state variables accurately enough that they may be considered to be known exactly. A more usual situation is that (a) only one or more functions of the state variables are measured, (b) these measurements contain random errors, (c) the process is driven by random disturbances. In this case the system no longer has a steady equilibrium state as assumed in Chapters 2 and 3. However, it may have a "statistical steady state" with a stationary probability density for the state and control variables.

If the process and the measurements can be modeled adequately as Gauss-Markov processes, the probability densities are completely specified by mean values and covariance matrices of the state and control variables.

We shall assume a stationary, linear model of the form

$$\dot{x} = Fx + Gu + \Gamma\eta \tag{4.1.1}$$

$$z = Hx + \nu \tag{4.1.2}$$

where z = measurement vector,

H = measurement distribution matrix,

η = process disturbance vector,

ν = measurement error vector.

We shall also assume that η and ν are independent purely random processes (white noise) with

$$\left.\begin{aligned}
&E[\eta(t)] = 0 \ , \\
&E[\eta(t)\eta^T(\tau)] = Q\,\delta(t-\tau) \ , \\
&E[\nu(t)] = 0 \ , \\
&E[\nu(t)\nu^T(\tau)] = R\,\delta(t-\tau) \ , \\
&E[\eta(t)\nu^T(\tau)] = 0 \ ,
\end{aligned}\right\} \tag{4.1.3}$$

where Q = process noise spectral density,

R = measurement noise spectral density.

4.2 - Observer-Controller by Pole Assignment

If the process and measurement noise are considered negligibly small (i.e. $Q \cong 0$, $R \cong 0$), it is possible to "reconstruct" the state using an "observer" (L-2), of the form

$$\dot{\hat{x}} = F\hat{x} + Gu + K(z-H\hat{x}) \tag{4.2.1}$$

where $\hat{x}$ = estimate of x . If the state x is observable with the measurements z , a gain matrix K can be found to assign the eigenvalues (poles) of the estimate error system arbitrarily, where

$$\dot{\tilde{x}} = (F-KH)\tilde{x} \ , \tag{4.2.2}$$

and $$\tilde{x} \triangleq \hat{x} - x = \text{estimate error.}$$

This estimate, $\hat{x}$, can then be used with the controller feedback gains, C , discussed in Section 2.2:

$$u = -C\hat{x} \tag{4.2.3}$$

The eigenvalues of the closed-loop observer-controller are the eigenvalues of $F-GC$ (the controller eigenvalues) plus the eigenvalues of $F-KH$ (the observer poles).

4.3 - Filter-Controller by Quadratic Synthesis

A useful technique for the design of control systems with additive white noise in the measurements and the process is quadratic synthesis coupled with optimal filtering. The system is modeled as a vector Gauss-Markov process (Eqns. (4.1.1)-(4.1.3), and the performance index is the expected value, given the measurement, of an integral quadratic penalty function:

$$\underset{u}{\text{Min}} J = \int_{t_o}^{\infty} E\left\{ (x^T A x + u^T B u) \,\middle|\, z \right\} dt \tag{4.3.1}$$

The minimizing solution has been shown (P-2) to be the optimal deterministic controller using the maximum likelihood estimates (Kalman filter, K-1) of the system states. Thus, $u=-C\hat{x}$ and

$$\dot{\hat{x}} = F\hat{x}+Gu+K(z-H\hat{x})$$

where $$C = B^{-1}G^{T}S \; ; \quad SF+F^{T}S+A-SGB^{-1}G^{T}S = 0 \tag{4.3.2}$$

and $$K = PH^{T}R^{-1} \; ; \quad FP+PF^{T}+\Gamma Q\Gamma^{T}-PH^{T}R^{-1}HP = 0$$

The steady covariance matrix of the state variables is given by

$$X \triangleq E\{xx^{T}\} = \hat{X} + P$$

where $$(F-GC)\hat{X} + \hat{X}(F-GC)^{T} + KRK^{T}=0 \tag{4.3.3}$$

P is the steady covariance matrix of the error in the estimates ($\tilde{x}$) which is uncorrelated with the estimate ($\hat{x}$).

The covariance matrix X is the stochastic analog of the deterministic steady response. The steady output covariance is a measure of how accurately the system is being controlled on the average. In fact, the performance index is a weighted covariance trade off between the state (or output) and the control.

4.4 - Example: Lateral Aircraft Autopilot

Consider again the lateral aircraft control problem. Suppose that instead of perfect knowledge of the state, three noisy measurements of roll attitude, yaw attitude, and lateral position are available. In addition, suppose the system is disturbed by gusty winds. The wind gust effects will be modeled as two independent Gaussian white noise processes: the first is due to lateral gusts, and the second is due to the lateral gradient of the vertical gusts. The power spectral densities of the process and measurement noise and the RMS values corresponding to a 50 ft correlation length along the flight path are given in Table 4.1.

Table 4.1

Noise Component	RMS	PSD
Lateral gusts	10 ft/sec	10000 ft^3/sec^2
Vertical gust gradient	10 ft/sec per 100 ft	1.0 ft/sec^2
Roll attitude error	0.5 deg	25.0 $ft\text{-}deg^2$
Yaw attitude error	0.5 deg	25.0 $ft\text{-}deg^2$
Lateral position error	10 ft	10000 ft^3

Use of the eigenvalue decomposition technique to solve for the steady filter and controller, yields the filter gains given in Appendix E. The closed-loop poles (in units of hft^{-1}) for the estimate error equation are shown in Fig. 8. The RMS estimation error and the RMS state response are shown in Table 4.2.

Table 4.2

	φ' deg/hft	φ deg	ψ' deg/hft	ψ deg	ϵ deg	y ft	δa deg	δr deg
RMS Error	0.872	0.651	0.366	0.472	0.169	3.09	~	~
RMS Response	1.47	3.31	0.500	0.979	0.651	7.43	6.30	3.17

The two poles near the origin correspond to the estimation of the lateral position by combining the attitude and position measurements. The relatively large time constant reflects the heavy filtering of the noise in the lateral position measurement. Also, the RMS roll response is considerably larger than the RMS yaw or path response. This is due to the use of banked turns as the primary controlling influence on the lateral position.

The RMS response in Table 4.2 can be compared to the RMS response shown in Table 4.3 where exact knowledge of the state is assumed.

Table 4.3

	φ' deg/hft	φ deg	ψ' deg/hft	ψ deg	ϵ deg	y ft	δa deg	δr deg
RMS Response	1.022	2.11	0.309	0.536	0.407	4.07	5.07	2.92

Chapter 5

Regulator Design for Constant Disturbances and Non-Zero Set Points Using Estimates of the State

5.1 - The Effect of Uncompensated Constant Disturbances

Consider the Gauss-Markov system model of Chapter 4 with the addition of a known constant disturbance vector, w:

$$\left.\begin{aligned} \dot{x} &= Fx + Gu + \Gamma_o w + \Gamma_1 \eta \ , \\ z &= Hx + \nu \ , \\ \dot{w} &= 0 \ . \end{aligned}\right\} \tag{5.1.1}$$

If the filter-controller of Section 4.3 is used for the system, a statistical steady state will still be reached but it will have a non-zero mean value:

$$\left.\begin{aligned} E[\tilde{x}] &\to (F-KH)^{-1}\Gamma_o w \ , \\ E[x] &\to (F-GC)^{-1}[GC\ (F-KH)^{-1}-I]\Gamma_o w \ , \\ E[u] &\to -C\{E[x] + E[\tilde{x}]\} \ . \end{aligned}\right\} \tag{5.1.2}$$

Suppose that the aircraft with the filter-controller of Section 4.4 is subjected to a 25 ft/sec steady crosswind. Since there is no direct compensation for a steady crosswind, it will go to steady offsets of -0.693 deg in roll, -4.38 deg in yaw, and 24.1 ft in lateral position. Such large offsets in yaw attitude and lateral position are unacceptable.

5.2 - Design with Exact Knowledge of the Disturbances

If the disturbances, w, are known exactly, this knowledge can be used in the filter for estimating the state and in the feed forward to the controls, i.e.

$$u = -C_x\hat{x} + C_y y_d - C_w w \ , \tag{5.2.1}$$

where

$$\dot{\hat{x}} = F\hat{x} + Gu + \Gamma_o w + K(z-H\hat{x}) \ . \tag{5.2.2}$$

The feedback gains C_x and K are determined as in Chapters 2 and 4,

as though $y_d=0$, $w=0$; the feed forward gains C_y and C_w are determined as in Section 3.2. This system eliminates the mean value offset described in Section 5.1.

5.3 - Design Using Estimates of the Disturbances

The constant disturbances are seldom known accurately so they must usually be estimated along with the state variables from the available measurements. However, the straightforward approach has a pitfall; by straightforward approach, we mean optimal estimation considering the disturbances as additional state variables with the random bias model,

$$\begin{aligned} &\dot{w} = 0 \ , \\ &E[w(t_o)] = 0 \ , \quad E[w(t_o)w^T(t_o)] = X_{ww} \ , \end{aligned} \tag{5.3.1}$$

i.e. the disturbances are constant, unpredictable, and vary from sample to sample. (In Section 5.1, the disturbances were assumed constant, known, and the same for all samples.) The controller with the augmented filter becomes

$$\begin{aligned} u &= -C_x\hat{x} + C_y y_d - C_w\hat{w} \ , \\ \dot{\hat{x}} &= F\hat{x} + Gu + \Gamma_o\hat{w} + K_1(z-H\hat{x}) \ , \\ \dot{\hat{w}} &= K_2(z-H\hat{x}) \end{aligned} \tag{5.3.2}$$

where $K_1 = P_{11}H^TR^{-1}$, $K_2 = P_{12}^TH^TR^{-1}$,

$$\begin{aligned} 0 &= FP_{11}+P_{11}F^T+\Gamma_1 Q\Gamma_1^T-P_{11}H^TR^{-1}HP_{11}+\Gamma_o P_{12}^T+P_{12}\Gamma_o^T \ , \\ 0 &= FP_{12}+\Gamma_o P_{22}-P_{11}H^TR^{-1}HP_{12} \ , \\ 0 &= P_{12}^TH^TR^{-1}HP_{12} \ . \end{aligned} \tag{5.3.3}$$

However, (5.3.3) indicates that $K_2=0$ since both P_{12} and $P_{22}=0$. This occurs because the constant disturbances, w, can be estimated exactly by a time-varying Kalman filter. The $K_2=0$ steady filter is useless for estimating the disturbances. Two alternatives to using a time-varying filter are (1) to use an exponentially-correlated model of the disturbances or (2) to use integral control. A random walk model of

the disturbances can also be used, but the steady covariance of the controlled state may be unbounded even though the filter is well defined.

Exponentially-Correlated Model of the Disturbances

A useful model of the disturbances is the exponentially-correlated model with a time constant, τ, long compared to characteristic times of the system being controlled:

$$\left.\begin{aligned} \dot{w} &= -\frac{1}{\tau} w + \eta_o , \\ E[\eta_o] = 0 , \quad E[\eta_o(t)&\eta_o^T(\tau)] = Q_o \, \delta(t-\tau) , \end{aligned}\right\} \tag{5.3.4}$$

where $\frac{\tau}{2} Q_o = \chi_{ww} = E[w(t)w^T(t)]$ = given constant.

This is essentially a slowly-changing bias model, which is more realistic than the "constant forever" bias model of (5.3.1). Here we must make a choice of τ , whereas in a random walk model we must make a choice of Q_o ; the choice of Q_o is usually more nebulous than the choice of τ .

The steady filter that results from this model is again given by (5.3.2) and (5.3.3) except that the last two equations in (5.3.3) are replaced by

$$\left.\begin{aligned} 0 &= FP_{12} - \frac{1}{\tau} P_{12} + \Gamma_o P_{22} - P_{11} H^T R^{-1} H P_{12} , \\ 0 &= \frac{2}{\tau} (P_{22} - \chi_{ww}) + P_{12}^T H^T R^{-1} H P_{12} \end{aligned}\right\} \tag{5.3.5}$$

and the second equation of (5.3.2) is replaced by

$$\dot{\hat{w}} = -\frac{1}{\tau} \hat{w} + K_2(z - H\hat{x}) . \tag{5.3.6}$$

With this filter, the system is predicted to reach a statistical steady state, and the usual techniques of predicting covariances may be used. Precisely speaking, the mean value offset of Section 5.1 will not be entirely removed since this filter will not estimate a truly constant disturbance exactly. However, for large τ the mean value offset will be negligibly small.

Use of the exponentially-correlated model to design a constant gain filter for estimating the lateral aircraft motions yields the filter gains shown in Appendix F where we assumed $V\tau$ = 12500 ft and X_{ww} = (25 ft/sec)^2 . The closed loop poles of the estimate error equations are those of Fig. 8 plus an additional pole at -0.3 hft^{-1}. The steady RMS response using this filter is shown in Table 5.1. In addition, the RMS response for w=0 and the steady response for a 25 ft/sec steady crosswind are shown for comparison purposes with the preceding and succeeding sections.

Table 5.1

	φ' deg/hft	φ deg	ψ' deg/hft	ψ deg	ϵ deg	y ft	δa deg	δr deg
RMS	1.595	4.51	0.635	1.297	0.713	8.15	10.12	13.04
RMS (w=0)	1.523	3.38	0.509	0.973	0.647	7.42	6.87	4.87
Offset	0.0	-2.71	0.0	-0.147	0.0	0.728	-7.45	12.18

The offsets for ψ and y are due to the small error in estimating the constant disturbance and are insignificant when compared to the fluctuating component in any given sample.

Filter Poles for the State Estimation System

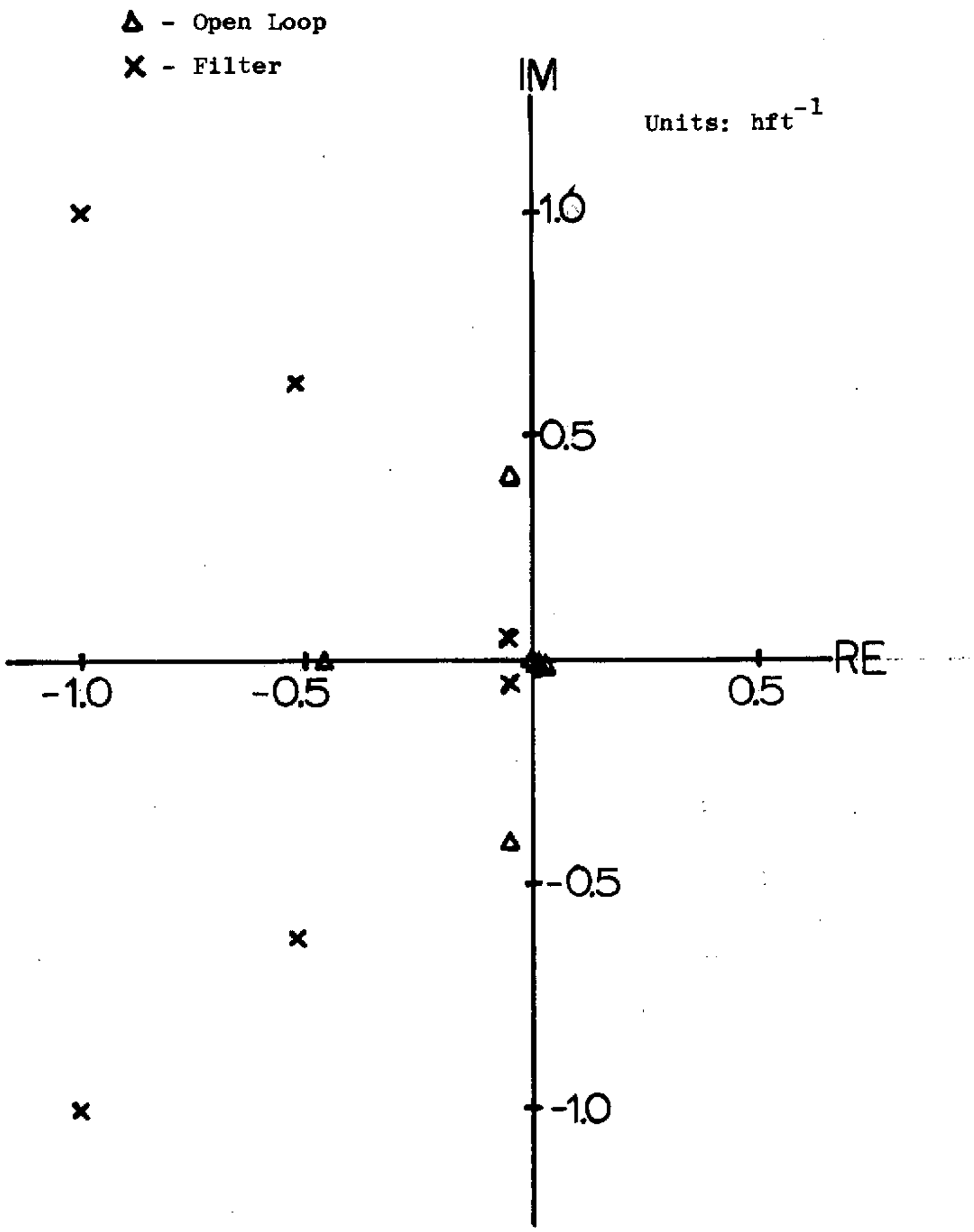


FIG. 8

5.4 - Design Using Modified Integral Control

Consider now the use of integral control with state estimation. Suppose a steady filter for the system has been designed assuming no constant disturbances:

$$\dot{\hat{x}} = F\hat{x} + Gu + K_1(z-H\hat{x}) \ . \tag{5.4.1}$$

we then introduce integral control in the following form (cf. Section 3.4):

$$u = -C_x\hat{x} + C_y y_d - C_v(v+L\hat{x}) \ , \tag{5.4.2}$$

where

$$\dot{v} = T\hat{x} - y_d + K_2(z-H\hat{x}) \ , \tag{5.4.3}$$

and C_x, C_y, C_v, L are chosen as in Section 3.4, while K_2 is a constant matrix yet to be chosen. It is straightforward to show that, for any K_2, the eigenvalues of the closed-loop system are the eigenvalues of $F-GC_x$ plus the eigenvalues of $F-K_1H$ plus the eigenvalues of $-LGC_v$, so that C_v can be chosen separately rather than simultaneously with C_x and K_1.

K_2 can be chosen to produce zero mean value offsets in the outputs. For w=constant it follows from (5.4.1) and (5.4.3) that

$$\left.\begin{aligned} E(\tilde{x}) &\rightarrow (F-K_1H)^{-1}\Gamma_o w \ , \\ E(Tx-y_d) &\rightarrow (K_2H-T)E(\tilde{x}) \end{aligned}\right\} \tag{5.4.4}$$

where $\tilde{x}=\hat{x}-x$=error in the estimate of x. Thus

$$E(Tx-y_d) \rightarrow (K_2H-T)(F-K_1H)^{-1}\Gamma_o w \ . \tag{5.4.5}$$

If K_2 can be chosen such that

$$(T-K_2H)(F-K_1H)\Gamma_o = 0 \ , \tag{5.4.6}$$

then the mean value offsets of the output will be zero.

For the important particular case where the outputs are measured, i.e.

$$z = Tx + \nu \ , \tag{5.4.7}$$

so that $T \equiv H$, it is obvious that (5.4.6) is satisfied if

$$K_2 = I = \text{identity matrix} . \tag{5.4.8}$$

The integrals of (5.4.3) become then simply

$$\dot{v} = z - y_d . \tag{5.4.9}$$

In general, the K_2 matrix can be chosen to insure zero mean offset if the rank of the matrix $[T(F-K_1H)^{-1}\Gamma_o]$ is equal to the rank of the augmented matrix $[T(F-K_1H)^{-1}\Gamma_o : H(F-K_1H)^{-1}\Gamma_o]$. Usually, this means that either the space spanned by T is included in that spanned by H or the dimension of the measurement is greater than or equal to the dimension of the steady disturbance. In the former case, it is possible to find K_2 so that $K_2H=T$, and the outputs (y) are measured directly with only white noise measurement error (ν). In the more general second case, at least as many independent measurements as independant constant disturbances are required. In this second case, the stationary performance is sensitive to modeling errors in F and Γ_o, whereas, in the first case when $K_2H=T$, the only sensitivity is to the measurement distribution H.

Expressions for the steady covariance matrix of x are given in Appendix G.

The above results are applied to the lateral aircraft control problem, using the undisturbed steady filter of Appendix E, the integral control law of Section 3.4, and the direct integration of the lateral position and yaw attitude measurements. The steady RMS response to the noisy gusts and measurements and the steady offsets for the constant 25 ft/sec crosswind are shown in Table 5.2.

Table 5.2

	φ' deg/hft	φ deg	ψ' deg/hft	ψ deg	ϵ deg	y ft	δa deg	δr deg
RMS	1.83	4.15	0.602	1.02	0.765	6.80	9.91	5.21
Offset	0	-2.71	0	0	0	0	-7.64	+12.5

Notice that the lateral position RMS response is slightly smaller than using the state-disturbance estimator at the cost of increased RMS aileron control.

Chapter 6

Conclusions

Several techniques for constant disturbance compensation have been presented. The concept of integral control was generalized to the multi-input, multi-output output case. A procedure for synthesizing the integral control gains separately from the standard regulator-filter was presented. Using this technique, a regulator-filter whose transient response is satisfactory can be augmented with integral control for improved steady behavior. The state estimate filter is left unchanged and the state feedback gains are modified in a straightforward manner.

The results were applied to the lateral aircraft control problem for automatic landing in the presence of a steady crosswind. It was found that state plus integral feedback provided an excellent technique for reducing the steady offset, without sacrificing good transient behavior. The use of a filter to estimate the state from noisy measurements coupled with the state plus integral feedback control laws resulted in acceptable RMS response and zero steady offset.

The integral control technique of constant disturbance compensation was compared to compensation using disturbance estimators. It was found that integral control amounts to a special form of estimation. However, the RMS response may be larger for the integral control formulation because no use is made of the assumed knowledge of the disturbance distribution as in the conventional estimator. This disadvantage of integral control must be weighed against the advantage that the steady offset remains zero even in the presence of modeling errors. The integral control technique does, however, exhibit sensitivity to bias or other long correlation disturbances in the measurements. Often, these measurement bias errors are not observable (as in the case of the lateral position measurement), and the conventional estimator does no better.

In conclusion, then, integral control is likely to be most successful in situations where there are system model uncertainties and the measurements are unbiased.

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Appendix A

State Dynamics Matrix for Lateral Motions of a DC-8 Aircraft in Landing Approach with Open Loop Eigensystem

ORDER OF SYSTEM = 6

OPEN LOOP DYNAMICS MATRIX....

-3.904D-01	0.0	2.502D-01	2.240D-01	-2.240D-01	0.0	φ'
1.000D 00	0.0	0.0	0.0	0.0	0.0	φ
-5.093D-02	0.0	-1.087D-01	-1.276D-01	1.276D-01	0.0	ψ'
0.0	0.0	1.000D 00	0.0	0.0	0.0	ψ
0.0	5.426D-02	0.0	4.572D-02	-4.572D-02	0.0	ϵ
0.0	0.0	0.0	0.0	1.745D 00	0.0	y

OPEN LOOP EIGENVALUES AND EIGENVECTORS........ (hft^{-1})

COMPLEX EIGENVALUE(1)........

(-0.04483468)+J(0.40648859)

COMPLEX EIGENVECTOR(1)........

```
( -0.04483468)+J(  0.40648859)
(  1.00000000)+J(  0.0       )
( -0.26250553)+J( -0.20808545)
( -0.43538417)+J(  0.69380997)
(  0.07822031)+J( -0.08434428)
( -0.39431806)+J( -0.29229680)
```

REAL EIGENVALUE (1)........

(0.0)+J(0.0)

REAL EIGENVECTOR(1)........

```
(  0.0       )
(  0.0       )
(  0.0       )
(  0.0       )
(  0.0       )
(  1.00000000)
```

REAL EIGENVALUE (2)........

(-0.46048882)+J(0.0)

REAL EIGENVECTOR(2)........

```
( -0.37782160)
(  0.82047942)
( -0.00841106)
(  0.01826551)
( -0.10934841)
(  0.41437050)
```

REAL EIGENVALUE (3)........

(0.00533819)+J(0.0)

REAL EIGENVECTOR(3)........

```
(  0.00000167)
(  0.00031376)
(  0.00001625)
(  0.00304391)
(  0.00305910)
(  0.99999064)
```

REAL EIGENVALUE (4)........

(-0.00000000)+J(0.0)

REAL EIGENVECTOR(4)........

```
(  0.00000000)
(  0.00000000)
(  0.00000000)
( -0.00000000)
( -0.00000000)
(  1.00000000)
```

Appendix B

Control Distribution Matrix and State Feedback Gains for Lateral Motions of a DC-8 Airplane in Landing Approach, with Closed-Loop Eigensystem

ORDER OF SYSTEM = 6

NUMBER OF CONTROLS = 2

OPEN LOOP DYNAMICS MATRIX....

-3.904D-01	0.0	2.502D-01	2.240D-01	-2.240D-01	0.0	φ'
1.000D 00	0.0	0.0	0.0	0.0	0.0	φ
-5.093D-02	0.0	-1.087D-01	-1.276D-01	1.276D-01	0.0	ψ'
0.0	0.0	1.000D 00	0.0	0.0	0.0	ψ
0.0	5.426D-02	0.0	4.572D-02	-4.572D-02	0.0	ε
0.0	0.0	0.0	0.0	1.745D 00	0.0	y

STATE COST MATRIX....

0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	2.500D-01	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	4.444D-03

THE CONTROL DISTRIBUTION MATRIX....

δa	δr
1.224D-01	-3.057D-02
0.0	0.0
8.971D-03	6.557D-02
0.0	0.0
0.0	-9.772D-03
0.0	0.0

THE CONTROL WEIGHTING MATRIX.......

1.000D-02	0.0
0.0	1.000D-02

EIGENSYSTEM OF OPTIMAL CLOSED LOOP SYSTEM.. (hft^{-1})

```
COMPLEX EIGENVALUE( 1)........          COMPLEX EIGENVECTOR( 1)........

( -0.32300031)+J(  0.50246618)          (  0.53588176)+J( -0.19436612)
                                        ( -0.75883202)+J( -0.57870318)
                                        ( -0.32300031)+J(  0.50246618)
                                        (  1.00000000)+J(  0.0        )
                                        (  0.00772588)+J( -0.02156565)
                                        ( -0.06519990)+J(  0.01508147)

COMPLEX EIGENVALUE( 2)........          COMPLEX EIGENVECTOR( 2)........

( -0.11996941)+J(  0.24002028)          (  0.20186374)+J( -0.04816881)
                                        ( -0.49691354)+J( -0.59265542)
                                        ( -0.01739570)+J(  0.00442558)
                                        (  0.04373718)+J(  0.05061481)
                                        ( -0.06875038)+J(  0.13754744)
                                        (  1.00000000)+J(  0.0        )

REAL EIGENVALUE    ( 1)........         REAL EIGENVECTOR( 1)........

( -0.45287098)+J(  0.0        )         ( -0.37032118)
                                        (  0.81771894)
                                        ( -0.00349654)
                                        (  0.00772084)
                                        ( -0.11067944)
                                        (  0.42646943)

REAL EIGENVALUE    ( 2)........         REAL EIGENVECTOR( 2)........

( -0.25151727)+J(  0.0        )         (  0.12740354)
                                        ( -0.50653995)
                                        ( -0.00854898)
                                        (  0.03398965)
                                        (  0.12155221)
                                        ( -0.84331625)
```

THE CONTROL GAINS ARE:

	φ'	φ	ψ'	ψ	ϵ	y
$-\delta_a$	3.57180771	2.47071429	4.63055479	1.60213745	8.54435546	0.65915493
$-\delta_r$	0.23302354	0.49602571	9.18713550	3.62741330	2.91641691	0.09957298

Appendix C

Steady-State Solutions when the Number of Outputs is Less than the Number of Control Inputs

The problem is to find x_s and u_s to minimize

$$J = \frac{1}{2}\,(x_s^T A x_s + u_s^T B u_s) \ , \tag{C.1}$$

subject to the steady-state constraints

$$0 = Fx_s + Gu_s + \Gamma w \ , \tag{C.2}$$

$$y_d = Tx_s \tag{C.3}$$

Adjoining (C.2) and (C.3) to (C.1) by Lagrange multipliers λ^T and μ^T respectively, it follows directly that the unique minimizing values of x_s and u_s can be obtained by solving the linear equations

$$\begin{bmatrix} F \ , & -GB^{-1}G^T \ , & 0 \\ -A \ , & -F^T \ , & T^T \\ T \ , & 0 \ , & 0 \end{bmatrix} \begin{bmatrix} x_s \\ \lambda \\ \mu \end{bmatrix} = \begin{bmatrix} -\Gamma_w \\ 0 \\ y_d \end{bmatrix} \ , \tag{C.4}$$

where

$$u_s = -B^{-1}G^T\lambda \ . \tag{C.5}$$

As an example, consider a steady banked turn at constant altitude for the DC-8 of Appendices A and B. Only the state variables ϕ', ϕ, ψ', and $\beta \overset{\Delta}{=} \epsilon - \psi$ are of interest in this case and it is clear that

$$\phi' = 0 \ , \ \psi' = \frac{\omega}{V} \ ,$$

where V is forward velocity and ω is the turn rate. The relevant constraint equations (with $w=0$) are therefore:

$$0 = \overbrace{\begin{bmatrix} 0 & , .2502 , & -.224 \\ 0 & , -.1087 , & .1276 \\ .05426 , & -1.000 , & -04572 \end{bmatrix}}^{F} \overbrace{\begin{bmatrix} \phi_s \\ \psi'_s \\ \beta_s \end{bmatrix}}^{x_s} + \overbrace{\begin{bmatrix} .1224 & , -.03057 \\ .008971 , & .06557 \\ 0 & , -.009772 \end{bmatrix}}^{G} \overbrace{\begin{bmatrix} \delta a_s \\ \delta r_s \end{bmatrix}}^{u_s} , \quad (C.6)$$

$$\frac{\omega}{V} = \overbrace{[0, 1, 0]}^{T} \begin{bmatrix} \phi_s \\ \psi'_s \\ \beta_s \end{bmatrix} , \text{ where } V \text{ is in units of hft/sec.} \quad (C.7)$$

For this case we take the weighting matrices of (C.1) as

$$A = 0 , \quad B = I . \quad (C.8)$$

Note there is only one output, $\frac{\omega}{V}$, while there are two control inputs (δa and δr). With these data, the choices of $\phi_s, \psi'_s, \beta_s, \delta a_s, \delta r_s$ that minimize (C.1) while satisfying (C.6) and (C.7), obtained by solving (C.4) and (C.5), are:

$$\begin{bmatrix} \phi_s \\ \psi'_s \\ \beta_s \end{bmatrix} = \begin{bmatrix} 19.21 \\ 1.000 \\ 0.9722 \end{bmatrix} \frac{\omega}{V} , \quad \begin{bmatrix} \delta a_s \\ \delta r_s \end{bmatrix} = \begin{bmatrix} -0.3127 \\ -0.1914 \end{bmatrix} \frac{\omega}{V} . \quad (C.9)$$

It is interesting to compare (C.9) with a "truly banked turn" where the aerodynamic side force is specified to be zero, i.e. $-.04572\beta_s - .009772\, \delta r_s = 0$ (there are now two outputs and two inputs). For this case it is easy to show from (3.2.7) that

$$\begin{bmatrix} \phi_s \\ \psi'_s \\ \beta_s \end{bmatrix} = \begin{bmatrix} 18.43 \\ 1.000 \\ -0.7335 \end{bmatrix} \frac{\omega}{V} , \quad \begin{bmatrix} \delta a_s \\ \delta r_s \end{bmatrix} = \begin{bmatrix} -2.529 \\ 3.431 \end{bmatrix} \frac{\omega}{V} . \quad (C.10)$$

For the zero sideslip turn

$$\begin{bmatrix} \varphi_s \\ \psi'_s \\ \beta_s \end{bmatrix} = \begin{bmatrix} 18.77 \\ 1.000 \\ 0.000 \end{bmatrix} \frac{\omega}{V} \quad , \quad \begin{bmatrix} \delta a_s \\ \delta r_s \end{bmatrix} = \begin{bmatrix} -1.576 \\ 1.873 \end{bmatrix} \frac{\omega}{V}$$

Notice that both the "truly banked" and zero sideslip turns require substantially larger control deflections than the control minimized turn. The steady side specific force, a, experienced by a passenger is given as

$$a = g(18.43\ \omega/V - \varphi_s) ,$$

where φ_s is in radians, ω in rad/sec, and V in hft sec^{-1}. This is equivalent to tilts from vertical of only 4.2% and 1.8% of the "truly banked" steady bank angle for the control minimized and zero sideslip turns, respectively.

Appendix D

Quadratic Synthesis of Integral Control Gains

An alternative method for determining state plus integral feedback control gains is to use quadratic synthesis (P-1, K-2, B-5). We temporarily neglect the constant process disturbances and non-zero setpoints and augment the system with the additional states (v) given by $\dot{v}=Tx$. Adding weighting on v in the performance index, yields an optimal regulator problem:

$$\min_u J = 1/2 \int_{t_o}^{\infty} (x^T A_1 x + v^T A_2 v + u^T B u) dt \tag{D.1}$$

Subject to

$$\begin{aligned} \dot{x} &= Fx + Gu & x(t_o) &= x_o \\ \dot{v} &= Tx & v(t_o) &= v_o \end{aligned} \tag{D.2}$$

where v_o is arbitrary. When the controllability conditions are satisfied, the solution is

$$u = -B^{-1}G^T S_{11} x - B^{-1} G^T S_{12} v \tag{D.3}$$

where

$$\begin{aligned} S_{11}F + F^T S_{11} - S_{11} G B^{-1} G^T S_{11} + A_1 + S_{12} T + T^T S_{12}^T = 0 \ , \\ (F - GB^{-1}G^T S_{11})^T S_{12} + T^T S_{22} = 0 \ , \\ \text{and} \qquad A_2 - S_{12}^T G B^{-1} G^T S_{12} = 0 \end{aligned} \tag{D.4}$$

To properly choose the initial conditions (v_o) , the performance index (D.1) will be minimized with respect to v_o (M-1). The performance index is given in terms of the initial conditions as

$$J = 1/2 \begin{bmatrix} x_o^T \ , & v_o^T \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} \begin{bmatrix} x_o \\ v_o \end{bmatrix} \tag{D.5}$$

Thus, the minimizing v_o is given by

$$v_o = -S_{22}^{-1} S_{12}^T x_o \qquad \text{(D.6)}$$

When this control law is implemented with a non-zero setpoint the only modification required for a zero-offset steady state is to let

$$\dot{v} = Tx - y_d \qquad \text{(D.7)}$$

However, the response to a setpoint change will be faster if feed-forward of the setpoint is also included. The resulting control law is thus,

$$u = -C_x x + C_y yd - C_v v \qquad \text{(D.8)}$$

Appendix E

Filter Gains for the Estimation of Lateral Motions from Noisy Attitude and Position Measurements with Gusty Wind Disturbances and the Eigensystem of the Estimation Error

ORDER OF SYSTEM = 6

NUMBER OF CONTROLS = 2

NUMBER OF OBSERVATIONS = 3

NUMBER OF PROCESS NOISE SOURCES = 2

OPEN LOOP DYNAMICS MATRIX....

-3.904D-01	0.0	2.502D-01	2.240D-01	-2.240D-01	0.0	φ'
1.000D 00	0.0	0.0	0.0	0.0	0.0	φ
-5.093D-02	0.0	-1.087D-01	-1.276D-01	1.276D-01	0.0	ψ'
0.0	0.0	1.000D 00	0.0	0.0	0.0	ψ
0.0	5.426D-02	0.0	4.572D-02	-4.572D-02	0.0	ϵ
0.0	0.0	0.0	0.0	1.745D 00	0.0	y

THE CONTROL DISTRIBUTION MATRIX....

δ_a	δ_r
1.224D-01	-3.057D-02
0.0	0.0
8.971D-03	6.557D-02
0.0	0.0
0.0	-9.772D-03
0.0	-0.0

STATE DISTURBANCE DISTRIBUTION MATRIX....

2.240D-01	-3.904D-01
0.0	0.0
-1.276D-01	-5.093D-02
0.0	0.0
4.572D-02	0.0
0.0	0.0

POWER SPECTRAL DENSITY - STATE NOISE....

5.537D 00	0.0
0.0	5.537D 00

MEASUREMENT SCALING MATRIX.....

0.0	1.000D 00	0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.000D 00	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.000D 00

POWER SPECTRAL DENSITY - MEASUREMENT NOISE..

2.500D-01	0.0	0.0
0.0	2.500D-01	0.0
0.0	0.0	1.000D 02

EIGENSYSTEM OF ESTIMATE ERROR EQUATION.....(hft^{-1})

COMPLEX EIGENVALUE(1)........

(-1.03865067)+J(1.01531494)

COMPLEX EIGENVECTOR(1)........

```
(  1.00000000)+J(  0.0        )
(  0.44759741)+J( -0.69319355)
( -0.00932262)+J(  0.06015641)
(  0.08193031)+J(  0.01099606)
(  0.03536834)+J( -0.01701054)
( -0.03751940)+J( -0.03386997)
```

COMPLEX EIGENVALUE(2)........

(-0.52597023)+J(0.61790966)

COMPLEX EIGENVECTOR(2)........

```
(  0.28232492)+J(  0.17562874)
(  0.23021303)+J(  0.02854616)
(  0.37325066)+J(  0.61866735)
(  1.00000000)+J(  0.0        )
( -0.13180058)+J( -0.16416696)
( -0.33152040)+J(  0.19161289)
```

COMPLEX EIGENVALUE(3)........

(-0.04887570)+J(0.04832768)

COMPLEX EIGENVECTOR(3)........

```
( -0.00305548)+J( -0.00348359)
( -0.00208325)+J( -0.00215971)
(  0.00431253)+J(  0.00491622)
(  0.00628471)+J(  0.00555069)
(  0.02577604)+J(  0.02682738)
(  1.00000000)+J(  0.0        )
```

THE COVARIANCE OF THE ESTIMATION ERROR

0.76068438	0.35913348	-0.01907670	0.04071315	0.03108149
0.35913348	0.42368247	-0.02072033	0.00773122	0.03185824
-0.01907670	-0.02072033	0.13382472	0.09957324	-0.04144187
0.04071315	0.00773122	0.09957324	0.22297129	-0.03374479
0.03108149	0.03185824	-0.04144187	-0.03374479	0.02865678
-0.04427190	0.00745114	-0.00535122	-0.06528608	0.26659257

-0.04427190
0.00745114
-0.00535122
-0.06528608
0.26659257
9.55581974

FILTER STEADY STATE GAINS......

φ_m	ψ_m	y_m
1.43653394	0.16285259	-0.00044272
1.69472986	0.03092487	0.00007451
-0.08288130	0.39829297	-0.00005351
0.03092487	0.89188514	-0.00065286
0.12743295	-0.13497917	0.00266593
0.02980456	-0.26114433	0.09555820

STATE RMS RESPONSE

φ'	1.46850863
φ	3.30690313
ψ'	0.49982089
ψ	0.97850027
ϵ	0.65134758
y	7.43165997

CONTROL RMS RESPONSE

δa	6.30470433
δr	3.17164437

Appendix F

Filter Gains for the Estimation of the Lateral Motions Including an Exponentially Correlated Wind Disturbance and the Eigensystem of the Estimation Error

ORDER OF SYSTEM = 7

NUMBER OF CONTROLS = 2

NUMBER OF OBSERVATIONS = 3

NUMBER OF PROCESS NOISE SOURCES = 3

OPEN LOOP DYNAMICS MATRIX....

-3.904D-01	0.0	2.502D-01	2.240D-01	-2.240D-01	0.0	2.240D-01	φ'
1.000D 00	0.0	0.0	0.0	0.0	0.0	0.0	φ
-5.093D-02	0.0	-1.087D-01	-1.276D-01	1.276D-01	0.0	-1.276D-01	ψ'
0.0	0.0	1.000D 00	0.0	0.0	0.0	0.0	ψ
0.0	5.426D-02	0.0	4.572D-02	-4.572D-02	0.0	4.572D-02	ϵ
0.0	0.0	0.0	0.0	1.745D 00	0.0	0.0	y
0.0	0.0	0.0	0.0	0.0	0.0	-8.000D-03	β_w

THE CONTROL DISTRIBUTION MATRIX....

δa	δr
1.224D-01	-3.057D-02
0.0	0.0
8.971D-03	6.557D-02
0.0	0.0
0.0	-9.772D-03
0.0	0.0
0.0	0.0

STATE DISTURBANCE DISTRIBUTION MATRIX....

2.240D-01	-3.904D-01	0.0
0.0	0.0	0.0
-1.276D-01	-5.093D-02	0.0
0.0	0.0	0.0
4.572D-02	0.0	0.0
0.0	0.0	0.0
0.0	0.0	1.000D 00

POWER SPECTRAL DENSITY - STATE NOISE....

5.537D 00	0.0	0.0
0.0	5.537D 00	0.0
0.0	0.0	5.536D-01

```
MEASUREMENT SCALING MATRIX.....

 0.0          1.0000D 00   0.0          0.0          0.0          0.0          0.0
 0.0          0.0          0.0          1.0000D 00   0.0          0.0          0.0
 0.0          0.0          0.0          0.0          0.0          1.0000D 00   0.0

POWER SPECTRAL DENSITY - MEASUREMENT NOISE..

 2.5000D-01   0.0          0.0
 0.0          2.5000D-01   0.0
 0.0          0.0          1.0000D 02

EIGENSYSTEM OF ESTIMATE ERROR EOUATION..... (hft^-1)

COMPLEX EIGENVALUE( 1)........          COMPLEX EIGENVECTOR( 1)........

( -1.03499481)+J(  1.01794641)          (  1.00000000)+J(  0.0        )
                                        (  0.46290254)+J( -0.66328835)
                                        ( -0.08238768)+J(  0.08456159)
                                        (  0.03737167)+J(  0.03425782)
                                        (  0.05420321)+J( -0.02589666)
                                        (  0.01927429)+J( -0.06113575)
                                        (  0.34924082)+J( -0.10904082)

COMPLEX EIGENVALUE( 2)........          COMPLEX EIGENVECTOR( 2)........

( -0.52025652)+J(  0.63865307)          ( -0.03538373)+J(  0.12658824)
                                        (  0.01560911)+J(  0.12575094)
                                        ( -0.59215569)+J(  0.07544230)
                                        ( -0.39157950)+J(  0.56039075)
                                        (  0.18589657)+J( -0.04030693)
                                        (  0.17708061)+J( -0.27307853)
                                        (  1.00000000)+J(  0.0        )

COMPLEX EIGENVALUE( 3)........          COMPLEX EIGENVECTOR( 3)........

( -0.04857749)+J(  0.04867963)          (  0.00003175)+J( -0.00008351)
                                        ( -0.00015894)+J( -0.00004096)
                                        (  0.00001112)+J(  0.00017144)
                                        (  0.00133603)+J(  0.00009723)
                                        (  0.02779542)+J(  0.02786137)
                                        (  1.00000000)+J(  0.0        )
                                        (  0.02574761)+J(  0.02717536)

REAL EIGENVALUE   ( 1)........          REAL EIGENVECTOR( 1)........

( -0.30023284)+J(  0.0        )         ( -0.11905073)
                                        ( -0.07429456)
                                        (  0.15231280)
                                        (  0.18072602)
                                        ( -0.05495609)
                                        ( -0.09008243)
                                        ( -0.95567314)
```

THE COVARIANCE OF THE ESTIMATION ERROR

0.79933060	0.37985823	-0.06643974	0.00063180	0.04932665
0.37985823	0.43549315	-0.04773003	-0.01646479	0.04233659
-0.06643974	-0.04773003	0.19623749	0.15611134	-0.06562507
0.00063180	-0.01646479	0.15611134	0.27880852	-0.05615926
0.04932665	0.04233659	-0.06562507	-0.05615926	0.03821684
0.00665403	0.03868896	-0.07677372	-0.14197009	0.29923830
0.30049013	0.15956794	-0.37077524	-0.32021174	0.14371967

0.00665403	0.30049013
0.03868896	0.15956794
-0.07677372	-0.37077524
-0.14197009	-0.32021174
0.29923830	0.14371967
9.78637978	0.41803514
0.41803514	2.49140724

FILTER STEADY STATE GAINS......

φ_m	ψ_m	y_m
1.51943293	0.00252720	0.00006654
1.74197261	-0.06585916	0.00038689
-0.19092014	0.62444535	-0.00076774
-0.06585916	1.11523408	-0.00141970
0.16934636	-0.22463705	0.00299238
0.15475583	-0.56788036	0.09786380
0.63827175	-1.28084698	0.00418035

Appendix G

The Steady Covariance Using Modified Integral Control

Using the equations given in Section 5.4, the estimates and estimate errors are found to satisfy the equations

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{v} \\ \dot{\tilde{x}} \end{bmatrix} = \begin{bmatrix} (F-GC_*) & , -GC_v & , -K_1H \\ T & , \ 0 & , -K_2H \\ 0 & , \ 0 & , (F-K_1H) \end{bmatrix} \begin{bmatrix} \hat{x} \\ v \\ \tilde{x} \end{bmatrix} + \begin{bmatrix} GC_y yd \\ -yd \\ -\Gamma_o w \end{bmatrix}$$

$$+ \begin{bmatrix} K_1 \\ K_2 \\ K_1 \end{bmatrix} \nu + \begin{bmatrix} 0 \\ 0 \\ -\Gamma_1 \end{bmatrix} \eta \qquad (G.1)$$

Since y_d and w are considered to be deterministic, they affect only the mean value. The steady covariance then satisfies

$$\begin{aligned} 0 &= (F-GC_*)X_{\hat{x}\hat{x}} + X_{\hat{x}\hat{x}}(F-GC_*)^T - GC_v X_{\hat{x}v}^T - X_{\hat{x}v}C_v^T G^T \\ 0 &= (F-GC_*)X_{\hat{x}v} - GC_v X_{vv} + X_{\hat{x}\hat{x}}T^T + K_1 R K_2^T \\ 0 &= TX_{\hat{x}v} + X_{\hat{x}v}^T T^T + K_2 R K_2^T \\ 0 &= (F-K_1H)X_{\tilde{x}\tilde{x}} + X_{\tilde{x}\tilde{x}}(F-K_1H)^T + K_1 R K_1^T + \Gamma_1 Q \Gamma_1^T \end{aligned} \qquad (G.2)$$

and $\quad X_{\hat{x}\tilde{x}} \ , \ X_{v\tilde{x}} = 0 \quad$ and $\quad X_{\tilde{x}\tilde{x}} = P$

where $\quad 0 = FP + PF^T - PH^T R^{-1} HP + \Gamma_1 Q \Gamma_1^T$

and $\quad K_1 = PH^T R^{-1}$

Thus, it is seen that the integral control law does not affect the steady performance of the filter, i.e. the estimate error is still P and the covariance between the estimates and the estimate errors remains zero. Only the steady covariance of the state estimates is changed by using the integral control law.

$$X_{xx} = X_{\hat{x}\hat{x}} + P$$

$$X_{uu} = C_* X_{\hat{x}\hat{x}} C_*^T + C_v X_{vv} C_v^T + C_* X_{\hat{x}v} C_v^T + C_v X_{\hat{x}v}^T C_*^T$$